

(Corrigé type)

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1 Solution(6 Pts)

$$1. \quad f(0) = 0 - \cos(1) = -0.5403 \quad f(2) = 2 - \cos(\frac{1}{3}) = 1.055 \quad (1 \text{ Pt})$$

$$f(0) \times f(2) < 0 \quad \text{alors : } \exists c \in [0, 2] / f(c) = 0 \quad (0.5 \text{ Pt})$$

2. .

$$\text{1er iter : } c = \frac{a+b}{2} = \frac{0+2}{2} = 1 \quad f(1) = 0.1224 > 10^{-1}, \quad f(1) \times f(0) < 0 \text{ alors : } c \in [0, 1] \quad (1.5 \text{ Pt})$$

$$\text{2eme iter } c = \frac{0+1}{2} = 0.5 \quad f(0.5) = -0.28587 > 10^{-1}, \quad f(0.5) \times f(1) < 0 \text{ alors : } c \in [0.5, 1] \quad (1.5 \text{ Pt})$$

$$\text{3eme iter } c = \frac{0.5+1}{2} = 0.75 \quad f(0.75) = -0.0911 < 10^{-1},$$

alors la solution approché est : $c = 0.75$ (1.5 Pt)

2 Solution(7 Pts)

$$\left(\begin{array}{ccc|c} 3 & 2 & 1 & 66 \\ 6 & 5 & 4 & 168 \\ 15 & 14 & 15 & 504 \end{array} \right) \quad (1pt)$$

1 : Pivot1 = 3

$$\left(\begin{array}{ccc|c} \frac{3}{3} & \frac{2}{3} & \frac{1}{3} & \frac{66}{3} \\ 6 & 5 & 4 & 168 \\ 15 & 14 & 15 & 504 \end{array} \right) = \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & 22 \\ 6 & 5 & 4 & 168 \\ 15 & 14 & 15 & 504 \end{array} \right) \quad (0.5pt)$$

2+3 : Pivot2 = 6 , et Pivot3 = 15

$$\left(\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & 22 \\ 6 - 6 \times 1 & 5 - 6 \times \frac{2}{3} & 4 - 6 \times \frac{1}{3} & 168 - 6 \times 22 \\ 15 - 15 \times 1 & 14 - 15 \times \frac{2}{3} & 15 - 15 \times \frac{1}{3} & 504 - 15 \times 22 \end{array} \right) = \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & 22 \\ 0 & 1 & 2 & 36 \\ 0 & 4 & 10 & 174 \end{array} \right) \quad (1pt)$$

4 : Pivot4 = 1 pas de changement

$$\left(\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & 22 \\ 0 & 1 & 2 & 36 \\ 0 & 4 & 10 & 174 \end{array} \right) \quad (0.5pt)$$

5+6 : Pivot5 = $\frac{2}{3}$ Pivot6 = 4

$$\left(\begin{array}{ccc|c} 1 & \frac{2}{3} - \frac{2}{3} \times 1 & \frac{1}{3} - \frac{2}{3} \times 2 & 22 - \frac{2}{3} \times 36 \\ 0 & 1 & 2 & 36 \\ 0 & 4 - 4 \times 1 & 10 - 4 \times 2 & 174 - 4 \times 36 \end{array} \right) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 36 \\ 0 & 0 & 2 & 30 \end{array} \right) \quad (1pt)$$

Pivot7=2 :

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 36 \\ 0 & 0 & 1 & 15 \end{array} \right) \quad (0.5pt)$$

Pivot8=-1, et pivot9=2 :

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 - (-1 \times 1) & -2 - (-1 \times 5) \\ 0 & 1 & 2 - 2 \times 1 & 36 - 2 \times 15 \\ 0 & 0 & 1 & 15 \end{array} \right) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 15 \end{array} \right) \quad (1pt)$$

Donc : $x_1 = 13$ $x_2 = 6$ $x_3 = 15$ (1.5 pt)

3 Solution(7 Pts)

$$I_s = \frac{h}{3} \left(f(x_1) + f(x_{n+1}) + 4 \sum_{i=2(pair)}^n f(x_i) + 2 \sum_{i=3(impair)}^{n-1} f(x_i) \right) \quad (1pt)$$

1 iter : $n = 2, h = 1$ (0.5pt)

x = 0 1 2
F = 1.0000 0.5000 0.2000

$I_1 = 1.0667$ (0.5+0.5pt)

2 iter : $n = 4, h = 0.5000$ (0.5pt)

x = 0 0.5000 1.0000 1.5000 2.0000
F = 1.0000 0.8000 0.5000 0.3077 0.2000

$I_2 = 1.1050$ (0.5+0.5pt)

3 iter : $n = 6, h = 0.3333$ (0.5pt)

x = 0 0.3333 0.6667 1.0000 1.3333 1.6667 2.0000
F = 1.0000 0.9000 0.6923 0.5000 0.3600 0.2647 0.2000

$I_3 = 1.1070$ (0.5+0.5pt)

$P = |I_3 - I_2| = |1.1070 - 1.1050| = 0.002 < 10^{-2}$, La précision est au rang de 10^{-2} (1.5pt)